

Bohr-like black holes

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Abstract

The idea that black holes (BHs) result in highly excited states representing both the “hydrogen atom” and the “quasi-thermal emission” in quantum gravity is today an intuitive but general conviction. In this paper it will be shown that such an intuitive picture is more than a picture. In fact, we will discuss a model of quantum BH somewhat similar to the historical semi-classical model of the structure of a hydrogen atom introduced by Bohr in 1913. The model is completely consistent with existing results in the literature, starting from the celebrated result of Bekenstein on the area quantization.

Researchers in quantum gravity [20] intuitively think that, in some respects, BHs are the fundamental bricks of quantum gravity in the same way that atoms are the fundamental bricks of quantum mechanics. This analogy suggests that the BH mass should have a discrete spectrum. In this extended abstract, we show that the such an intuitive picture is more than a picture. Starting from the natural correspondence between Hawking radiation [1] and BH quasi-normal modes (QNMs) [2–4], we show that QNMs can be really interpreted in terms of BH quantum levels discussing a BH model somewhat similar to the semi-classical Bohr model of the structure of a hydrogen atom [5, 6].

One considers Dirac delta perturbations [2–4, 7] representing subsequent absorptions of particles having negative energies which are associated to emissions of Hawking quanta in the mechanism of particle pair creation. BH responses to perturbations are QNMs [2–4, 8–12, 21], which are frequencies of radial spin- j perturbations obeying a time independent Schrödinger-like equation [2–4, 12].

They are the BH modes of energy dissipation which frequency is allowed to be complex [2–4, 12]. For large values of the principal quantum number n , where $n = 1, 2, \dots$, QNMs become independent of both the spin and the angular momentum quantum numbers [2–4, 8, 12, 13, 14], in perfect agreement with *Bohr's Correspondence Principle* [15], which states that “transition frequencies at large quantum numbers should equal classical oscillation frequencies”. In other words, Bohr's Correspondence Principle enables an accurate semi-classical analysis for large values of the principal quantum number n , i.e, for excited BHs. By using that principle, Hod has shown that QNMs release information about the area quantization as QNMs are associated to absorption of particles [13, 44]. Hod's work was refined by Maggiore [8] who solved some important problems. On the other hand, as QNMs are *countable* frequencies, ideas on the *continuous* character of Hawking radiation did not agree with attempts to interpret QNMs in terms of emitted quanta, preventing to associate QNMs to Hawking radiation [12]. Recently, ourselves and collaborators [2–4, 8–11, 21] observed that the non-thermal spectrum of Parikh and Wilczek [16] also implies the countable character of subsequent emissions of Hawking quanta. This issue enables a natural correspondence between QNMs and Hawking radiation, permitting to interpret QNMs also in terms of emitted energies [2–4, 8–11]. In fact, Dirac delta perturbations due to discrete subsequent absorptions of particles having negative energies, which are associated to emissions of Hawking quanta in the mechanism of particle pair creation by quantum fluctuations, generates BH QNMs [2–4, 8–11]. On the other hand, the correspondence between emitted radiation and proper oscillation of the emitting body is a fundamental behavior of every radiation process in science. Based on such a natural correspondence between Hawking radiation and BH QNMs, one can consider QNMs in terms of quantum levels also for emitted energies [2–4, 8–11]. For large values of the principal quantum number n , i.e, for excited BHs, and independently of the angular momentum quantum number, the QNMs expression of the Schwarzschild BH which takes into account the non-strictly thermal behavior of the radiation spectrum is obtained as [2–4]

$$\omega_n = a + ib + \frac{in}{4M - 2|\omega_n|} \simeq \frac{in}{4M - 2|\omega_n|}, \quad (1)$$

where a and b are real numbers with $a = \frac{\ln 3}{4\pi(2M - |\omega_n|)}$, $b = \frac{1}{4(2M - |\omega_n|)}$ for $j = 0, 2$ (scalar and gravitational perturbations), $a = 0$, $b = 0$ for $j = 1$ (vector perturbations) and $a = 0$, $b = \frac{1}{4(2M - |\omega_n|)}$ for half-integer values of j . On the other hand, as $a, b \ll |\frac{in}{4M - 2|\omega_n|}|$, a fundamental consequence is that the quantum of area obtained from the asymptotic values of $|\omega_n|$ is an intrinsic property of Schwarzschild BHs because for large n the leading asymptotic behavior of $|\omega_n|$ is given by the leading term in the imaginary part of the complex frequencies, and it does not depend on the spin content of the perturbation [2–4, 8]. An intuitive derivation of eq. (1) can be found in [3, 4]. We *rigorously* derived such an equation in the Appendix of [2]. If one considers the leading asymptotic behavior, the physical solution for the absolute values of the frequencies (1) is [2–4]

$$E_n \equiv |\omega_n| = M - \sqrt{M^2 - \frac{n}{2}}. \quad (2)$$

E_n is interpreted like the total energy emitted by the BH at that time, i.e. when the BH is excited at a level n [2–4]. Considering an emission from the ground state (i.e. a BH which is not excited) to a state with large $n = n_1$ and using eq. (2), the BH mass changes from M to [2–4]

$$M_{n_1} \equiv M - E_{n_1} = \sqrt{M^2 - \frac{n_1}{2}}. \quad (3)$$

In the transition from the state with $n = n_1$ to a state with $n = n_2$ where $n_2 > n_1$ the BH mass changes again from M_{n_1} to

$$\begin{aligned} M_{n_2} &\equiv M_{n_1} - \Delta E_{n_1 \rightarrow n_2} = M - E_{n_2} \\ &= \sqrt{M^2 - \frac{n_2}{2}}, \end{aligned} \quad (4)$$

where

$$\Delta E_{n_1 \rightarrow n_2} \equiv E_{n_2} - E_{n_1} = M_{n_1} - M_{n_2} = \sqrt{M^2 - \frac{n_1}{2}} - \sqrt{M^2 - \frac{n_2}{2}}, \quad (5)$$

is the jump between the two levels due to the emission of a particle having frequency $\Delta E_{n_1 \rightarrow n_2}$. Thus, in our BH model [2], during a quantum jump a discrete amount of energy is radiated and, for large values of the principal quantum number n , the analysis becomes independent of the other quantum numbers. In a certain sense, QNMs represent the "electron" which jumps from a level to another one and the absolute values of the QNMs frequencies represent the energy "shells" [2]. In Bohr model [5, 6] electrons can only gain and lose energy by jumping from one allowed energy shell to another, absorbing or emitting radiation with an energy difference of the levels according to the Planck relation (in standard units) $E = hf$, where h is the Planck constant and f the transition frequency. In our BH model [2], QNMs can only gain and lose energy by jumping from one allowed energy shell to another, absorbing or emitting radiation (emitted radiation is given by Hawking quanta) with an energy difference of the levels according to eq. (5). The similarity is completed if one notes that the interpretation of eq. (2) is of a particle, the "electron", quantized on a circle of length [3]

$$L = \frac{1}{T_E(E_n)} = 4\pi \left(M + \sqrt{M^2 - \frac{n}{2}} \right), \quad (6)$$

which is the analogous of the electron travelling in circular orbits around the hydrogen nucleus, similar in structure to the solar system, of Bohr model [5, 6]. On the other hand, Bohr model is an approximated model of the hydrogen atom with respect to the valence shell atom model of full quantum mechanics. In the same way, our BH model should be an approximated model with respect to the definitive, but at the present time unknown, BH model arising from a full quantum gravity theory.

Let us discuss *the area quantization*. Setting $n_1 = n - 1$, $n_2 = n$ in eq. (5) on gets the emitted energy for a jump among two neighboring levels [2, 3, 4]

$$\Delta E_{n-1 \rightarrow n} = \sqrt{M^2 - \frac{n+1}{2}} - \sqrt{M^2 - \frac{n}{2}}. \quad (7)$$

An enlightening analysis that we rigorously developed in [2] shows that eq. (7) leads to the area quantum

$$|\Delta A_n| = |\Delta A_{n-1}| = 8\pi, \quad (8)$$

which is exactly the famous result of Bekenstein on the area quantization [17], and this *cannot* be a coincidence. Other fundamental results are: i) the famous formula of Bekenstein-Hawking entropy [1, 18, 19] reads [2]

$$(S_{BH})_{n-1} \equiv \frac{A_{n-1}}{4} = 8\pi N_{n-1} M_{n-1} \cdot \Delta E_{n-1 \rightarrow n} = 4\pi \left(M^2 - \frac{n-1}{2} \right) \quad (9)$$

before the emission and

$$(S_{BH})_n \equiv \frac{A_n}{4} = 8\pi N_n M_n \cdot \Delta E_{n-1 \rightarrow n} = 4\pi \left(M^2 - \frac{n}{2} \right), \quad (10)$$

after the emission, respectively; ii) the total BH entropy becomes [2]

$$\begin{aligned} (S_{total})_{n-1} &= 4\pi \left(M^2 - \frac{n-1}{2} \right) \\ &\quad - \ln \left[4\pi \left(M^2 - \frac{n-1}{2} \right) \right] + \frac{3}{32\pi \left(M^2 - \frac{n-1}{2} \right)} \end{aligned} \quad (11)$$

before the emission, and

$$\begin{aligned} (S_{total})_n &= 4\pi \left(M^2 - \frac{n}{2} \right) \\ &\quad - \ln \left[4\pi \left(M^2 - \frac{n}{2} \right) \right] + \frac{3}{32\pi \left(M^2 - \frac{n}{2} \right)} \end{aligned} \quad (12)$$

after the emission, respectively. Thus, both the Bekenstein-Hawking entropy and the total BH entropy results a function of the BH excited state n . We stress that our results are in perfect agreement with existing results in the literature, see [2-4] for details.

Conclusion remarks

We have shown that the intuitive but general conviction that BHs result in highly excited states representing both the “hydrogen atom” and the “quasi-thermal emission” in quantum gravity is more than a picture as we have indeed discussed a model of quantum BH somewhat similar to the historical semi-classical model of the structure of a hydrogen atom introduced by Bohr in 1913. This Bohr-like model of BHs is totally consistent with existing results in the literature, starting from the famous result of Bekenstein on the area quantization. The issue that semi-classical BHs are the analogous of the Bohr model for the hydrogen atom is also an intriguing starting point for future work on a new approach to quantum gravity based on the “electron” represented by QNMs, i.e. for the potential construction of a “QNMs quantum gravity”.

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